Calculating the height your rocket will reach.

Up to this point we would use the following equation to calculate the height reached by an accelerating object:

$$y = \frac{1}{2}at^2 + v_0 t + y_0$$

We can still use this equation but since the acceleration is not constant we must first write it as a function of time. Using Newton's Second Law we know: $a = \dfrac{Fnet}{m}$. The variability of the acceleration comes from the fact that F_{net} and m both change with time. Let's look first at the mass. The mass of your rocket will change because the motor ejects gasses as it burns. From the data for the motor tests given at the end of this document we see that the motor loses 7.4g in the 0.86s that it burns, so it loses mass at a rate of 0.0086 kg/s. If m_0 is the mass of the rocket at launch then the mass of the rocket at any time during the engine burn is given by:

$$m = m_0 - 0.0086 \frac{kg}{s} t$$

The net force on the rocket can be found by considering the effects of the earth's gravity (W), the motor thrust (T) and air drag (D). In general the equation would be $F_{net} = -W + T - D$ where the positive direction is chosen as up. The force due to earth's gravity is given by W = mg, as usual, but since m is a function of time it becomes:

$$W = 9.8 \frac{N}{kg}(m_0 - 0.0086 \frac{kg}{s} t)$$

The value for the thrust of the rocket motor can be found in two ways, both of which use the data from the motor tests. The first method is to use the total impulse measured by the force probe which, on average, was 4.341 N-s. Since this was delivered over a 0.86s time interval the *average* thrust of the motor is:

$$\overline{F} = \frac{\overline{J}}{t} = \frac{4.341NS}{0.86s} = 5.048N$$

Of course this isn't the *most* accurate value, but is simple to use. Naturally, the thrust is 0N after 0.86s when the motor burns out. The second method is to just copy and paste the force values from the LoggerPro file that were collected during a motor test. The advantage is that they are very close to the actual values of thrust for each time interval during the engine burn. Since the net force will affect several other calculations is may be advantageous to reduce the error as much as possible here. Since the data was collected every 0.02 seconds instead of every 0.01 seconds the values can be averaged to fill in the missing times. I have done this for you for one of the motor tests. It is in a spreadsheet that is linked to the data section of this document. Just double-click on the Excel icon below the thrust graph.

Finally, we need to be able to determine the air friction, or drag, on the rocket since this is a huge factor in how high it will go. Drag can be calculated using the following equation:

$$D = \frac{1}{2}\rho A C_d v^2$$

Where D is drag, ρ is the density of the air, A is the cross-sectional area of the rocket, C_d is the coefficient of drag for the rocket and v is the rockets velocity. The density of the air, ρ, is fairly constant for the small change in altitudes we are dealing with and is about 1.29 $\frac{kg}{m3}$.

To find the cross-sectional area of your rocket you find the area of the cross section of the body tube and each of the fins and add them together. The diameter of the tubes are known or easily measured so the area can be found with a=π r^2. To find the cross section of the fins measure the distance they extend out from the body tube, x, as shown in the diagram. Multiply this by the thickness of the fins. This is the cross section of one fin; multiply that by the number of fins for the total fin cross sectional area. Note that x is not necessarily the length of the edge of a fin but is the maximum extension of the fin from the body tube.
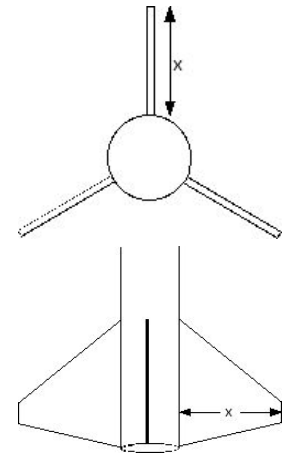
The coefficient of drag, C_d, is a dimensionless quantity (not unlike the familiar μ from our friction problems) and is usually found through testing in a wind tunnel. It can be estimated mathematically but it is a complicated calculation. For our purposes, and without any better information, we will estimate it to be 0.75.

Finally, v is the velocity of the rocket and must be calculated for each time interval using:
$$v = a\Delta t + v_0.$$

OK, so here it gets a bit sticky. The acceleration, a, we are trying to calculate using $a = \dfrac{Fnet}{m}$,

But v is a factor in this calculation since it affects D, which affects F_{net}. So, how do we resolve this circular reasoning? Remember, we have divided our flight into many "slices", each lasting 0.01 seconds so that things don't change much during one time interval. The velocity we are looking for is the final velocity of this time interval, say from t=0.22s to 0.23s. We can use the acceleration form the *previous* time interval as a good approximation of the acceleration during this time interval to calculate the new final velocity. The initial velocity for this time interval, v_0, is just the final velocity from the previous. For example, suppose the acceleration during the time interval t=0.21s and t=0.22s was 12$^m/_{s2}$ and the final velocity was 12 $^m/_s$. Then the final velocity at t=0.23s would be:

$$v = 12 \; ^m\!/_{s^2} (0.01s) + 12 \, ^m\!/_s = 12.12 \, ^m\!/_s$$

So, now we can put it all together and calculate F_{net}, which allows us to calculate the acceleration. Once we have the acceleration for each time interval we can use it to calculate the position of the rocket at the end of each time interval. When the positions quit increasing our rocket has reached its maximum height. This would be a nightmare if we had to do it all by hand. Fortunately we have a powerful tool to help us, a spreadsheet.

Data

Estes B6-4 rocket engine

time	thrust
0	0.030025
0.02	0.32917
0.04	0.867632
0.06	1.465922
0.08	2.423187
0.1	3.769341
0.12	5.235153
0.14	7.030025
0.16	8.58558
0.18	9.692418
0.2	10.29071
0.22	10.44028
0.24	10.41037
0.26	8.675324
0.28	7.32917
0.3	6.760794
0.32	6.461649
0.34	5.983016
0.36	5.833444
0.38	5.594127
0.4	5.474469
0.42	5.444555
0.44	5.354811
0.46	5.324897
0.48	5.265068
0.5	5.175324
0.52	5.115495
0.54	5.08558
0.56	4.995837
0.58	4.936008
0.6	4.965922
0.62	4.965922
0.64	4.906093
0.66	4.906093
0.68	4.906093
0.7	4.846264
0.72	4.876179
0.74	4.81635
0.76	3.350538
0.78	1.765068
0.8	0.65823
0.82	0.299256
0.84	0.149683
0.86	0.149683

B6-4 rocket test 1

Integral for: Latest | Force
Integral: -4.295 N*s

Δx: 0.86 Δy: 0.44
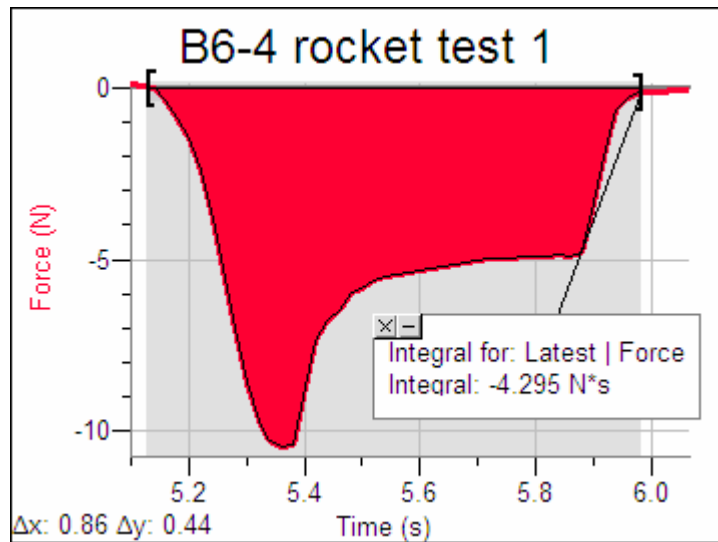
- Mass of new motor: 17.1 g
- Mass of used motor: 9.7 g

- Average burn time: 0.86s

- Average impulse: 4.341 N-s

Microsoft Excel
Worksheet